The Shape of the Correlation Function

Jakub Cimerman [1] Christopher Plumberg [2] Boris Tomášik [1, 3]

[1]Czech Technical University, FNSPE, Prague, Czech Republic

[2]Lund University, Lund, Sweden

[3]Matej Bel University, FNS, Banská Bystrica, Slovakia

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Introduction

- Correlation femtoscopy has become a standard technique for the experimental analysis of heavy-ion collisions
- Two-particle correlation functions are often fitted by Gaussian
- However, it seems that the real shape is not Gaussian
- The shape is often better reproduced by Lévy stable distribution
- It has been suggested that Lévy shape with specific exponent may identify the critical point
- We check if the observed shape can be caused by non-critical phenomena

HBT formalism

- Correlation function is defined as ratio of two-particle spectrum and one-particle spectra

$$C(p_1,p_2) = \frac{P_2(p_1,p_2)}{P(p_1)P(p_2)} = \frac{E_1 E_2 \frac{\mathrm{d}^6 N}{\mathrm{d}p_1^3 \mathrm{d}p_2^3}}{\left(E_1 \frac{\mathrm{d}^3 N}{\mathrm{d}p_1^3}\right)\left(E_2 \frac{\mathrm{d}^3 N}{\mathrm{d}p_2^3}\right)}$$

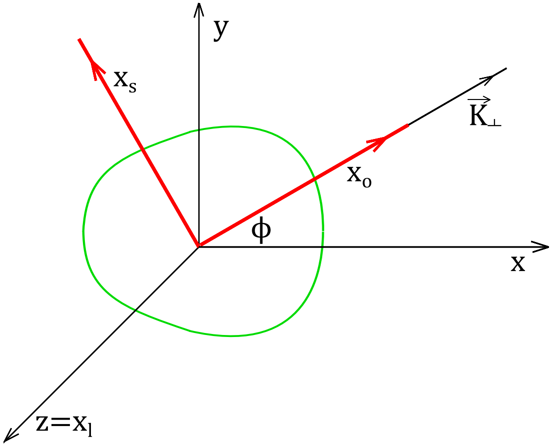
- We use correlation function in the form

$$C(q,K) - 1 \approx \frac{\left|\int \mathrm{d}^4 x S(x,K) \exp(iqx)\right|^2}{\left(\int \mathrm{d}^4 x S(x,K)\right)^2}$$

  - $K = \frac{1}{2}(p_1 + p_2)$, $q = p_1 - p_2$

Coordinate system

- *out, side, long* coordinate system



Parametrization of correlation function

- Gaussian parametrization

$$C_G(\vec{q}, \vec{K}) = 1 + \lambda \exp\left[-\sum_{i,j=o,s,l} R_{ij}^2 q_i q_j\right]$$

- Lévy parametrization

$$C_L(\vec{q}, \vec{K}) = 1 + \lambda' \exp\left[-\left|\sum_{i,j=o,s,l} R_{ij}'^2 q_i q_j\right|^{\alpha/2}\right]$$

- One-dimensional Lévy parametrisation

$$C_L(Q) = 1 + \lambda' \exp(-|R'Q|^{\alpha})$$

- Lévy index characterizes the shape of the correlation function
  - $\alpha = 2 \Rightarrow$ Gauss $\qquad \alpha = 1 \Rightarrow$ exponential

Effects leading to non-Gaussianities

- Ensemble averaging
  - Each event is different
  - Averaging over many events may affect the shape
- One-dimensional projection
- $\vec{K}$ averaging
- Resonance decays

$$C(q,K) \approx 1 + \frac{\left\langle \left| \int \mathrm{d}^4 x S(x,K) \exp(iqx) \right|^2 \right\rangle_{ev}}{\left\langle \left(\int \mathrm{d}^4 x S(x,K) \right)^2 \right\rangle_{ev}}$$

Effects leading to non-Gaussianities

- Ensemble averaging
- One-dimensional projection
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- Resonance decays

– Correlation function as a function of a scalar quantity

– Lorentz-invariant Q

$$Q_{LI}^2 = -q^\mu q_\mu$$

– Longitudinally boost-invariant Q

$$Q_{LBI}^2 = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{long,LCMS}^2}$$

Effects leading to non-Gaussianities

- Ensemble averaging
- One-dimensional projection
- $\vec{K}$ averaging
- Resonance decays

– The size of a bin in $\vec{K}$ cannot be taken arbitrarily small

– Correlation function must be averaged over some pair momentum interval

$$C(q,K) \approx 1 + \frac{\int_{bin} \mathrm{d}^3K \left| \int \mathrm{d}^4x S(x,K) \exp(iqx) \right|^2}{\int_{bin} \mathrm{d}^3K \left(\int \mathrm{d}^4x S(x,K) \right)^2}$$

Effects leading to non-Gaussianities

- Ensemble averaging
- One-dimensional projection
- $\vec{K}$ averaging
- Resonance decays
  - Different resonances contributes with different lengthscales and timescales
  - Correlation function therefore must deviate from a Gaussian form

Blast-Wave model

- This theoretical model is characterized by the emission function

$$S(x,p)\mathrm{d}^4x = \frac{m_t \cosh(\eta - Y)}{(2\pi)^3}\mathrm{d}\eta\mathrm{d}x\mathrm{d}y\frac{\tau\mathrm{d}\tau}{\sqrt{2\pi}\Delta\tau}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)\exp\left(-\frac{E^*}{T}\right)\Theta\left(1-\overline{r}\right)$$

Cooper-Frye prefactor

- Spatial anisotropy describes shape of the fireball

$$R(\theta) = R_0\left(1 - \sum_{n=2}^{\infty} a_n \cos\left(n(\theta - \theta_n)\right)\right)$$

- Flow anisotropy describes distribution of transverse rapidity

$$\rho(\overline{r},\theta_b) = \overline{r}\rho_0\left(1 + \sum_{n=2}^{\infty} 2\rho_n \cos\left(n(\theta_b - \theta_n)\right)\right)$$

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Gaussian smearing in proper time

- Spatial anisotropy describes shape of the fireball

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Boltzmann thermal distribution

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transverse box profile

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$E^* = p_\mu u^\mu$ - energy in the local rest frame

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$\overline{r} = \frac{r}{R(\theta)}$ - dimensionless transverse coordinate

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$$\eta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right) \qquad \tau = \sqrt{t^2 - z^2}$$

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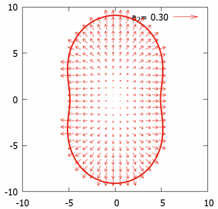

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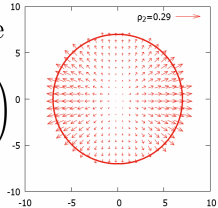


$$\rho(\overline{r},\theta_b) = \overline{r}\rho_0\left(1 + \sum_{n=2}^{\infty} 2\rho_n \cos\left(n(\theta_b-\theta_n)\right)\right)$$

DRAGON

[Comp. Phys. Comm. 180 (2009) 1642]

- Monte Carlo event generator
- Based on the Blast-Wave model with added resonance decays
- For this study we generated 50,000 events with parameters

$$\begin{array}{cc} T = 120\,\mathrm{MeV} & R = 7\,\mathrm{fm} \\ \tau_{fo} = 10\,\mathrm{fm}/c & \rho_0 = 0.8 \\ a_2 \in (-0.1; 0.1) & \rho_2 \in (-0.1; 0.1) \end{array}$$

- To generate correlation functions we used **CRAB** [S. Pratt]

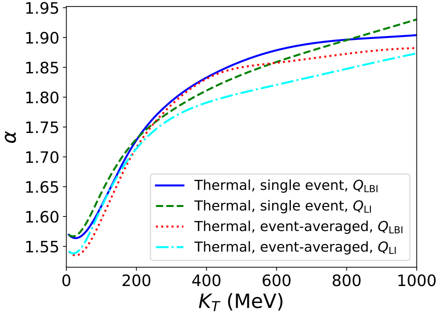
Hydrodynamics

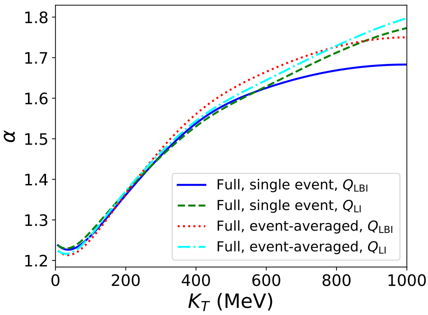
- Hydrodynamical model **iEBE-VISHNU** [Comp. Phys. Comm. 199 (2016) 61]
  - 2+1D hydrodynamic simulation
  - Israel-Stewart viscous hydrodynamics
  - Glauber MC initial conditions
- Extension to HBT using **HoTCoffeeh** [Phys. Rev. C 98 (2018) 034910]
  - calculates event-by-event correlation functions directly from Cooper-Frye integrals
  - includes all resonances
  - no hadron phase nor hadron cascades
- For this study we generated 1,000 events with parameters

$$0-10\%\ \text{Au+Au collisions at } 200A\ \text{GeV}$$
$$T_{fo} = 120\,\text{MeV} \qquad \eta/s = 0.08$$

Comparison of different non-Gaussian effects

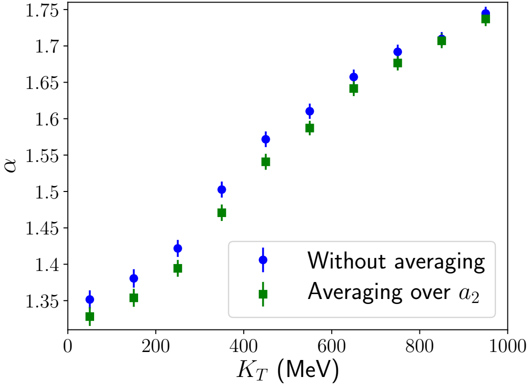
- Results of hydrodynamical approach
- Comparison of:
  thermal vs. full   single-event vs. event-averaged Q_{LBI} vs. Q_{LI}

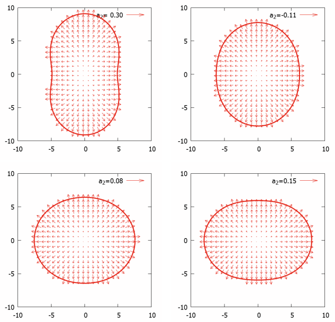




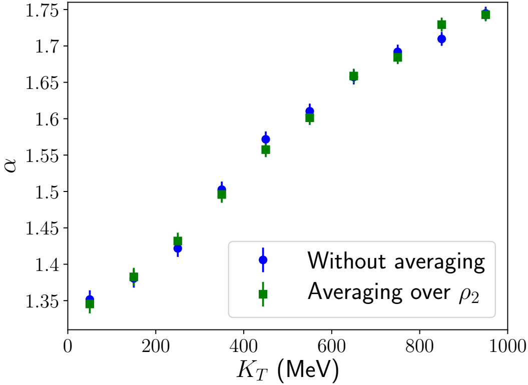
Event-by-event fluctuations

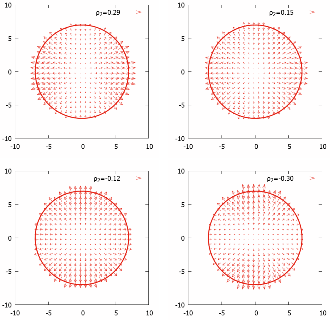
- Study of averaging over events - influence of spatial anisotropies
- Unaveraged - $a_2 = 0.05$ Averaged - $a_2 \in (-0.1; 0.1)$



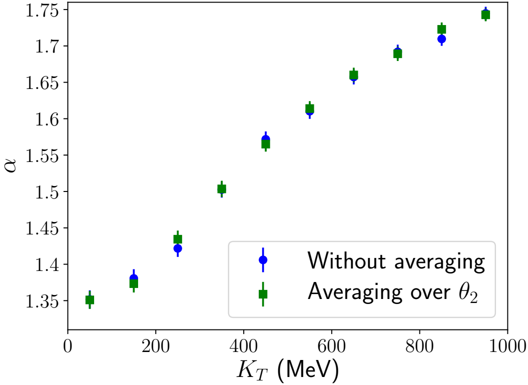


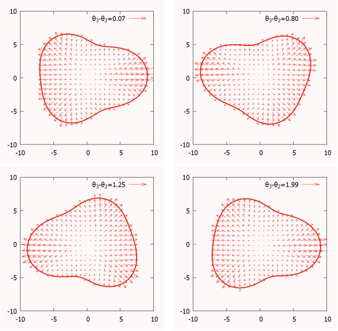
- Study of averaging over events - influence of flow anisotropies
- Unaveraged - $\rho_2 = 0.05$ Averaged - $\rho_2 \in (-0.1; 0.1)$



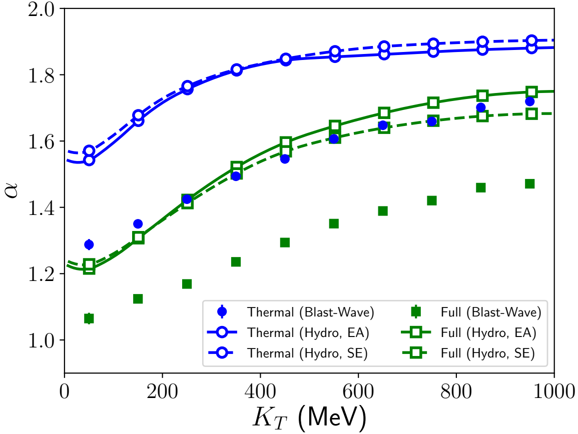


- Study of averaging over events - influence event plane direction
- Unaveraged - $\theta_2 = 0$ Averaged - $\theta_2 \in (0;\pi)$





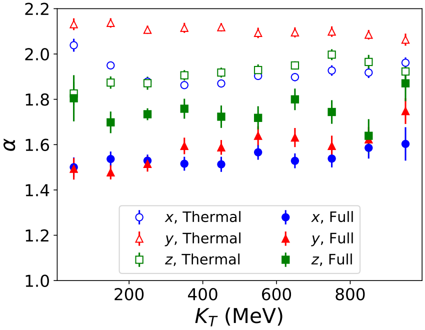
- Resonance decays push down the value of α by 0.2 for both models



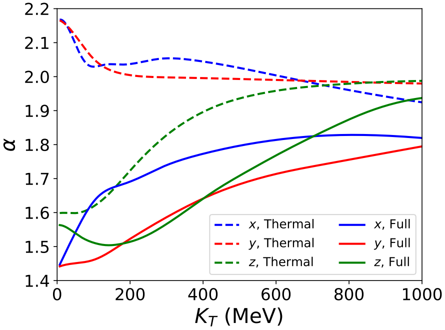
Correlation function in three dimensions

- 3D correlation function fitted via 1D Lévy function in each direction separately

Blast-Wave

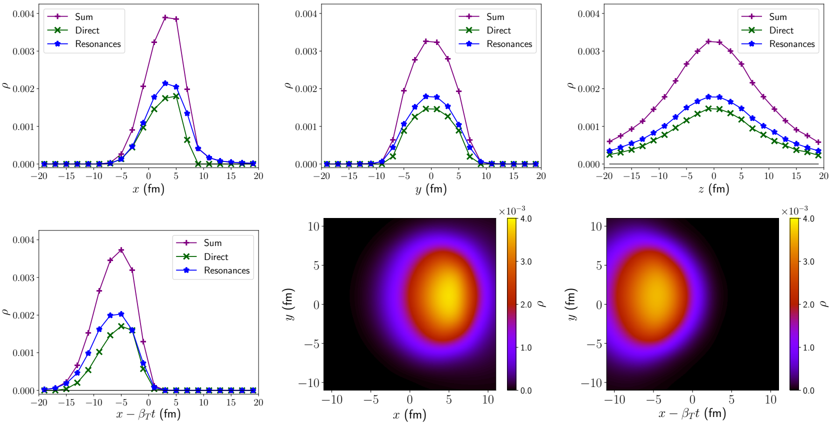


Hydro



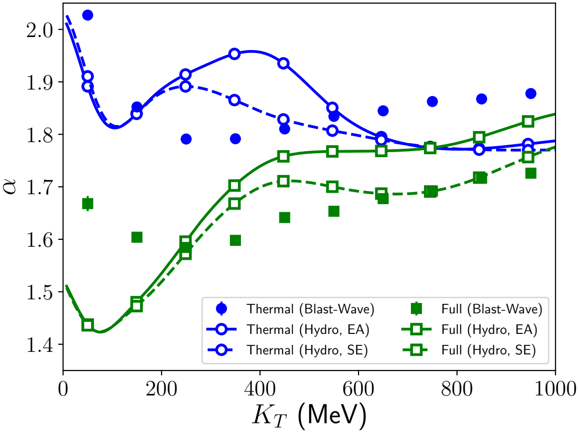
Profiles of emission sources

- To understand the differences in different directions we check the shape of the source which emits pions
- Pions are taken with $K_T \in (300; 400)$ MeV



3D fit to correlation function

- 3D Lévy fit to 3D correlation function



Lévy expansion

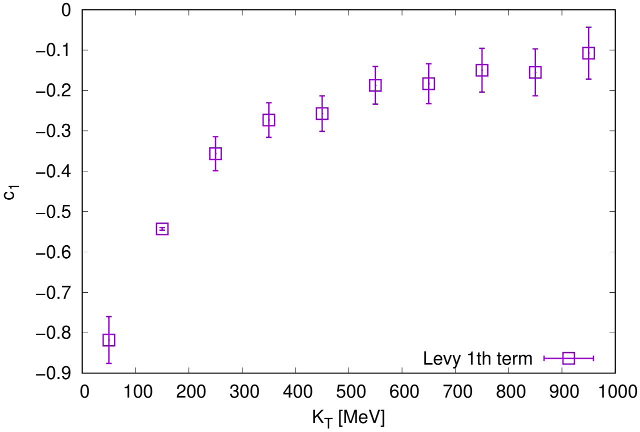
- Even Lévy parametrisation cannot describe our correlation functions perfectly
- To get the corrections to higher orders, we decompose the data into Lévy expansion up to 1st order

$$C(Q) = 1 + \lambda e^{-R^{\alpha}Q^{\alpha}} \left[1 + c_1 L_1(Q|\alpha)\right]$$

  - L_1 is Lévy polynomial, c_1 is complex Lévy coefficient
- However, such fits are very unstable

Lévy expansion

- 3D Lévy fit to 3D correlation function   Blast-Wave



- The first-order corrections are not negligible ⇒ our correlation functions are neither Lévy-shape

Conclusions

- Index of the Lévy-stable parametrization fitted to the correlation function may be influenced by a variety of different mechanisms
- It deviates substantially from the value of 2
- The most significant deviations arise from
  - projection from 3D relative momentum $\vec{q}$ to scalar Q
  - resonance decays
- These conclusions are model-independent