Non-Gaussian Sources and the Shape of the Correlation Function[1]

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1. Motivation

Correlation femtoscopy has become a standard technique for measuring and probing the space-time evolution of heavy-ion collisions. Usually, two-particle correlation functions are fitted to a Gaussian form. However, the real shape of the correlation function is often strongly non-Gaussian and better described by a Lévy-stable distribution. A Lévy index much below 2 has recently been observed experimentally [2]. It has been suggested that an even lower value of the Lévy index equal to 0.5 may identify matter produced at the critical endpoint of the QCD phase diagram [3]. Despite this, there are non-critical effects which can also influence the value of the Lévy index significantly, and it is crucial to quantify the magnitudes of these effects before assigning physical significance to a measurement of the Lévy index.

2. HBT Formalism

The two-particle correlation function probes the momentum-space structure of correlations between pairs of particles produced in heavy-ion collisions. The correlation function is often expressed in terms of the momentum difference $q = p_1 - p_2$ and the average momentum $K = \frac{1}{2}(p_1 + p_2)$. Using the smoothness approximation, the correlation function takes the form

$$C(q,K) - 1 \approx \frac{\left|\int \mathrm{d}^4x S(x,K)\exp(iqx)\right|^2}{\left(\int \mathrm{d}^4x S(x,K)\right)^2}. \tag{1}$$

The commonly used Gaussian parametrisation of this correlation function reads

$$C_G(\vec{q},\vec{K}) = 1 + \lambda(\vec{K})\exp\left[-\sum_{i,j=o,s,l} R^2_{ij}(\vec{K})q_iq_j\right], \tag{2}$$

where $R^2_{ij}(\vec{K})$ are the HBT radii characterizing the size of the homogeneity region.
Since the Gaussian parametrisation does not adequately describe the experimentally measured correlation function, we use the Lévy parametrisation

$$C_L(\vec{q},\vec{K}) = 1 + \lambda'(\vec{K})\exp\left[-\left|\sum_{i,j=o,s,l} R'^2_{ij}(\vec{K})q_iq_j\right|^{\alpha/2}\right] \tag{3}$$

where the Lévy index α controls the form of the distribution used to approximate the correlation function: for $\alpha = 2$, C_L is a Gaussian, while for $\alpha = 1$, it is an exponential distribution.

3. Effects Leading to Non-Gaussianities

We studied four effects which can lead to a non-Gaussian shape of the correlation function:

- **event averaging** - properties of events (such as size, geometric and dynamical anisotropies, and so on) tend to fluctuate randomly from one event to the next. Therefore averaging over these events may affect the shape of the correlation function.
- **one-dimensional projection** - in order to improve statistical precision one can use a one-dimensional projection of the relative momentum. The correlation function is then a function of a single scalar quantity. There are two ways to perform this projection:
  - Lorentz-invariant variable
  $$Q^2_{\mathrm{inv}} = -q^\mu q_\mu = \vec{q}\cdot\vec{q} - (q^0)^2, \tag{4}$$
  - longitudinally boost-invariant variable
  $$Q^2_{\mathrm{LCMS}} = \sqrt{(p_{1x}-p_{2x})^2 + (p_{1y}-p_{2y})^2 + \frac{(p_{1z}E_2 - p_{2z}E_1)^2}{K_0^2 - K_l^2}}. \tag{5}$$
- **averaging with respect to the pair momentum** $\vec{K}$ - when measuring the correlation function, bins in $\vec{K}$ must be created which cannot be taken arbitrarily small.
- **resonance decays** - different resonances introduce multiple scales into the correlation function, while the Gaussian is characterized by only a single lengthscale.

4. Models

In order to test the model independence of our conclusions, we considered two different models for our analysis.
The first one is the blast-wave model [4], which describes an expanding locally thermalised fireball. It also contains spatial and flow anisotropies. To generate events we use DRAGON [5, 6], which is a Monte Carlo event generator based on the blast-wave model with added resonance decays. For this study we generated sets of 50,000 events with parameters set to: temperature $T = 120\,\mathrm{MeV}$, the average transverse radius $R_0 = 7\,\mathrm{fm}$, freeze-out time $\tau_{fo} = 10\,\mathrm{fm}/c$, the strength of the transverse expansion $\rho_0 = 0.8$, second order spatial anisotropy $a_2 \in (-0.1; 0.1)$ and second order flow anisotropy $\rho_2 \in (-0.1; 0.1)$. To calculate correlation functions from these events we used CRAB [7].
The second model we used is a hydrodynamic model of the collision system using the boost-invariant iEBE-VISHNU event-generator [8, 9] with MC Glauber initial conditions. We generated 1,000 events of $0-10\%$ Au+Au collisions at $200A\mathrm{GeV}$ with a freeze-out temperature $T = 120\mathrm{MeV}$ and $\eta/s = 0.08$. To compute the HBT correlation functions we used the `HoTCoffeeh` code [10], which directly evaluates Cooper-Frye integrals with resonance-decay effects over the freeze-out surface on an event-by-event basis.

5. Results

First, we used the hydrodynamic model to check the relative importance of several of the effects discussed above. In Figure 1 we see the impact of three of these:

- correlation function with resonances (right panel) vs. without resonances (left panel),
- single event (solid blue and dashed green) vs. event-averaged (dotted red and dash-dotted cyan),
- Q_{inv} (solid blue and dotted red) vs. Q_{LCMS} (dashed green and dash-dotted cyan).

By far, the largest effect on $\alpha(K_T)$ is caused by resonance decays. Event averaging and the type of 1D projection used make almost no qualitative difference to the results.

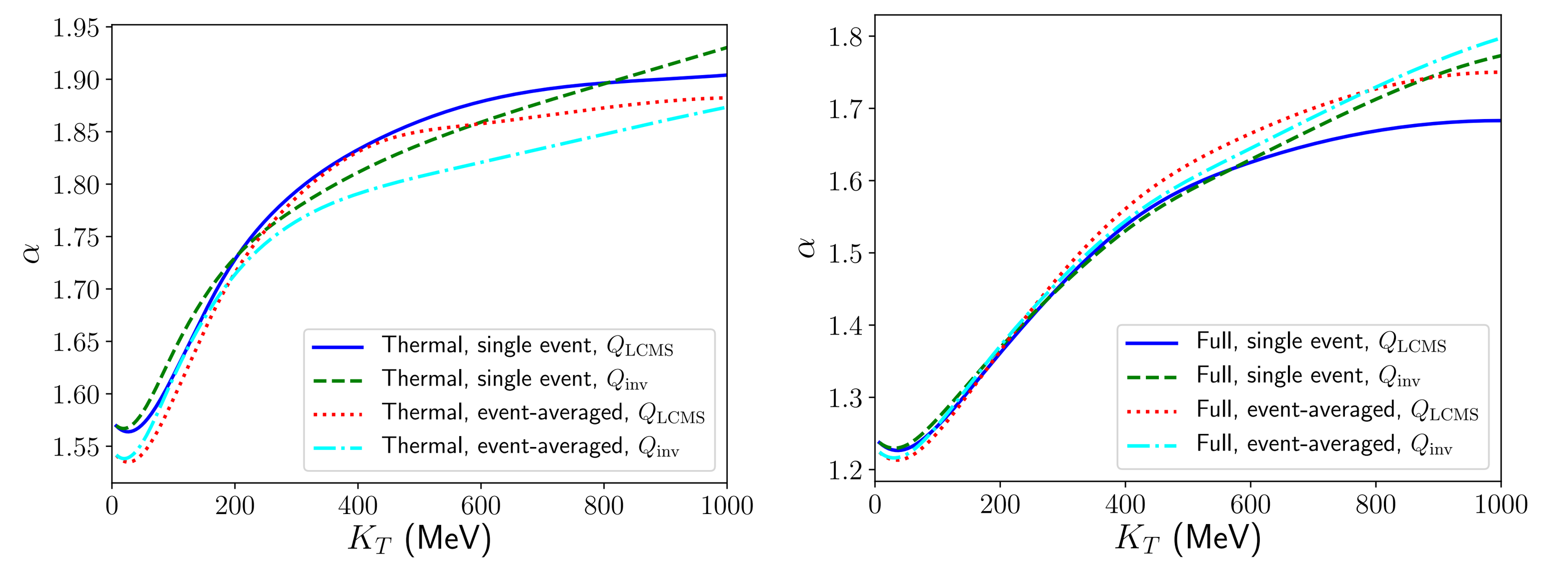

Figure 1: *A comparison of $\alpha(K_T)$ with and without different non-Gaussian effects in our hydrodynamic model.*

The left panel of Figure 2 shows the influence of averaging over the parameter of spatial anisotropy of the BW model. The effect of averaging is smaller than the error bars. Moreover, we obtained even smaller differences for averaging over ρ_2 and θ_2. Thus we can say that this effect plays no role in the resulting value of the Lévy index.
The right panel of Figure 2 shows the model-independent impact of resonances on the Lévy index. Both models predict that resonances reduce the value of the Lévy index by $\sim 0.2 - 0.3$.

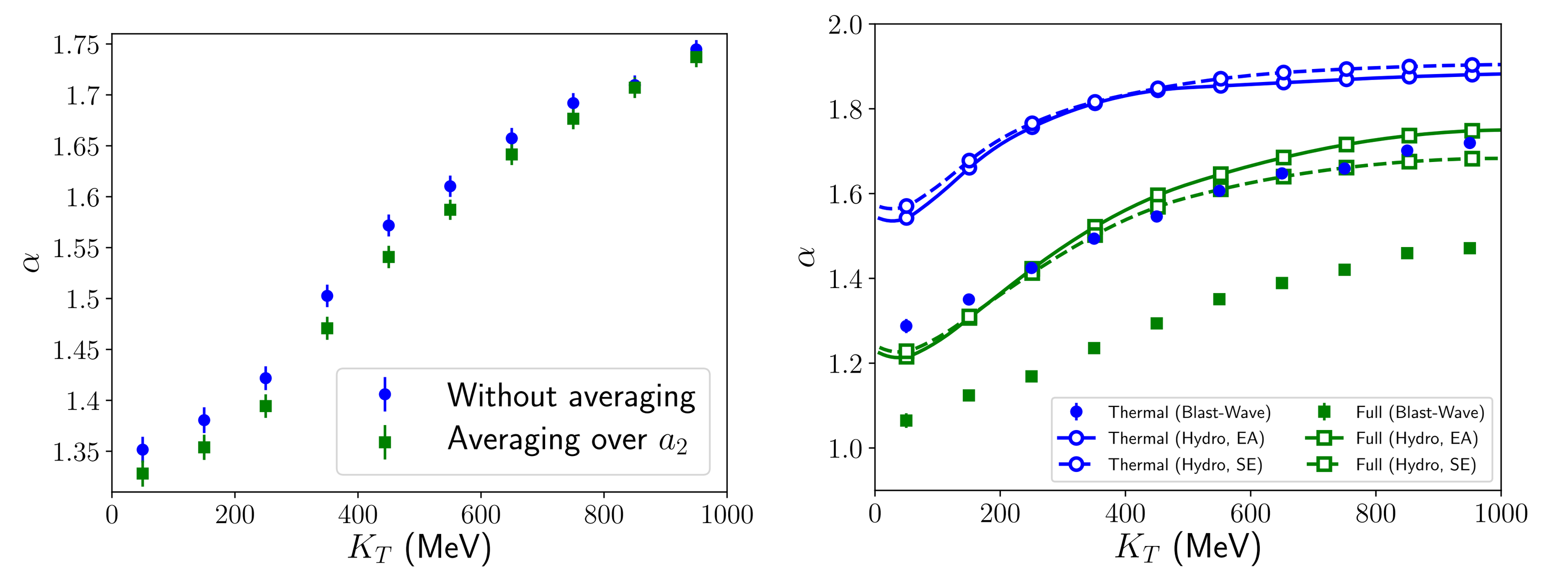

Figure 2: *Left: The effect of averaging over a_2 in the BW model. Right: Comparison of resonance effects in both models.*

To find out why the 1D projection affects the Lévy index so significantly, we have to look at the 3D correlation function. Figure 3 shows 1D fits to the correlation function in each direction separately. These plots show that, while the correlation function behaves similarly in the outward and sideward directions, the K_T-dependence in the longitudinal direction behaves differently. Moreover, it seems that the resonances do not affect the correlation function in the longitudinal direction as much as in the transverse one.

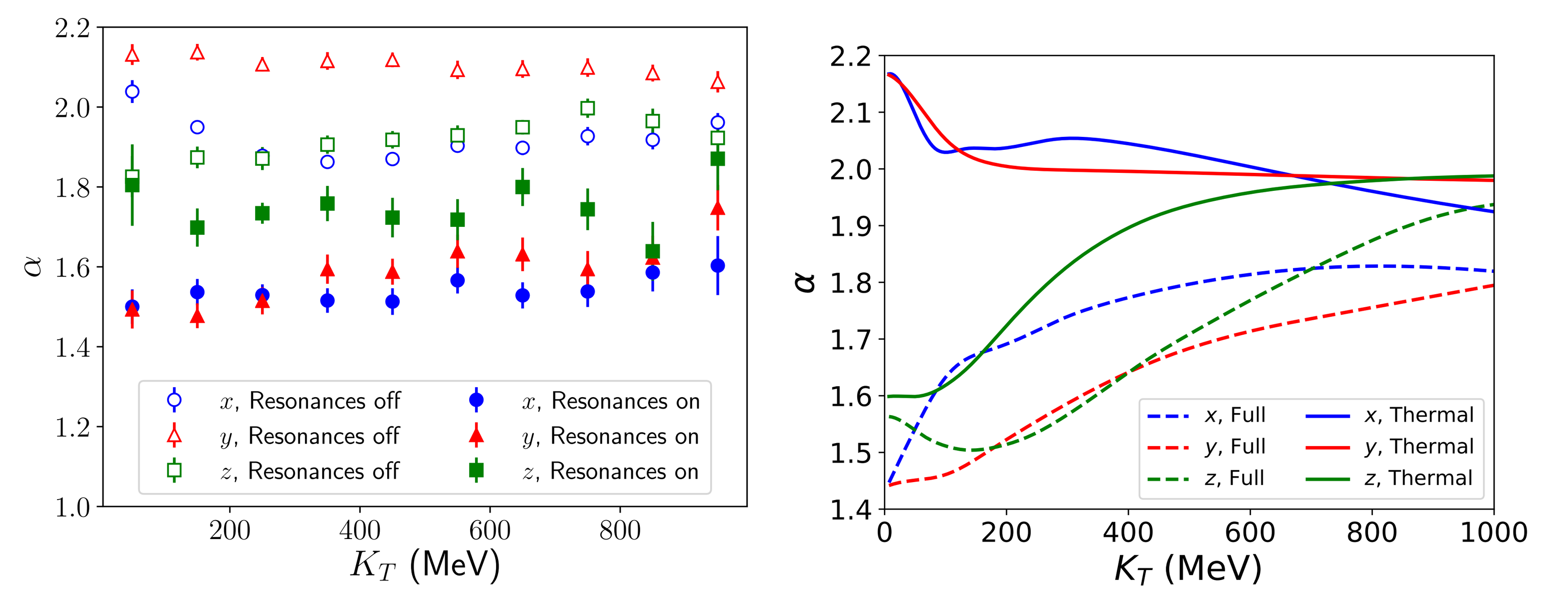

Figure 3: *The Lévy index of the 1D fits to the correlation function in $\vec{q}$ along different axes for the BW (left) and hydrodynamic (right) models.*

6. Conclusions

We have shown that the shape of the correlation function, as well as the value of the Lévy index, may be influenced by a variety of different mechanisms. Our results show that the Lévy index may deviate substantially from the value of 2 due to non-critical effects. The two most significant deviations arise, first, from the projection of the 3D relative momentum $\vec{q}$ onto a scalar Q, and second, from the inclusion of resonance decays. Since we used two different models, these results appear to be robust and not merely artifacts of the models we have used. For this reason, the conclusions presented here should be regarded as model-independent.

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