Higher-order anisotropies in the Blast-Wave Model

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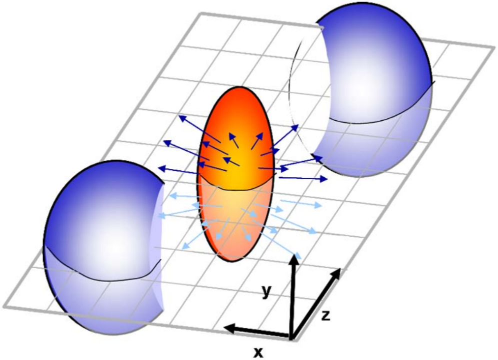
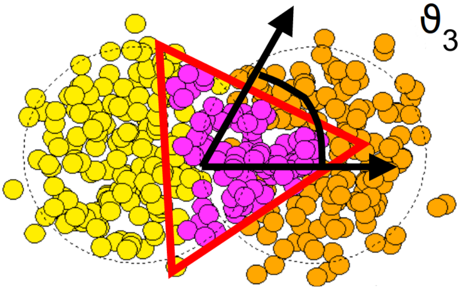
Anisotropy





Alver, Glasma Workshop 2010

Emission function of Blast-wave model

- $$S(x,p)\mathrm{d}^4x = \frac{m_t\cosh(\eta - Y)}{(2\pi)^3}\mathrm{d}\eta\mathrm{d}x\mathrm{d}y\frac{\tau\mathrm{d}\tau}{\sqrt{2\pi}\Delta\tau}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)\exp\left(-\frac{E^*}{T}\right)\Theta(1-\bar{r})$$

- $\eta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$ - spacetime rapidity
- $\tau = \sqrt{t^2 - z^2}$ - longitudinal proper time
- $E^* = p_\mu u^\mu$ - energy in rest frame
- $\bar{r} = \frac{r}{R(\theta)}$ - scaled radius of fireball in transverse plane

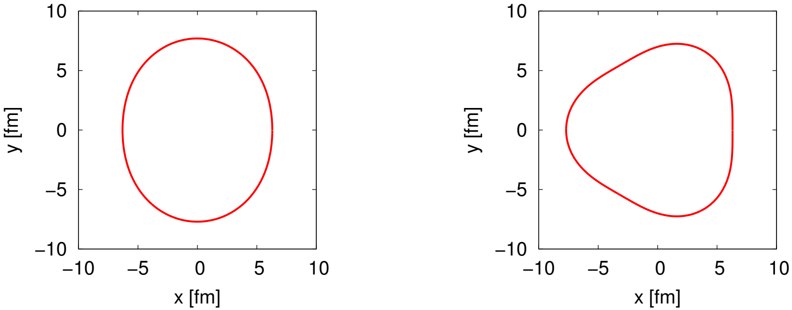
- $\frac{m_t\cosh(\eta - Y)}{(2\pi)^3}$ - Cooper-Frye prefactor
- $\frac{\tau\mathrm{d}\tau}{\sqrt{2\pi}\Delta\tau}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$ - Gaussian smearing in proper time
- $\exp\left(-\frac{E^*}{T}\right)$ - Boltzmann thermal distribution
- $\Theta(1-\bar{r})$ - box profile

Space anisotropy

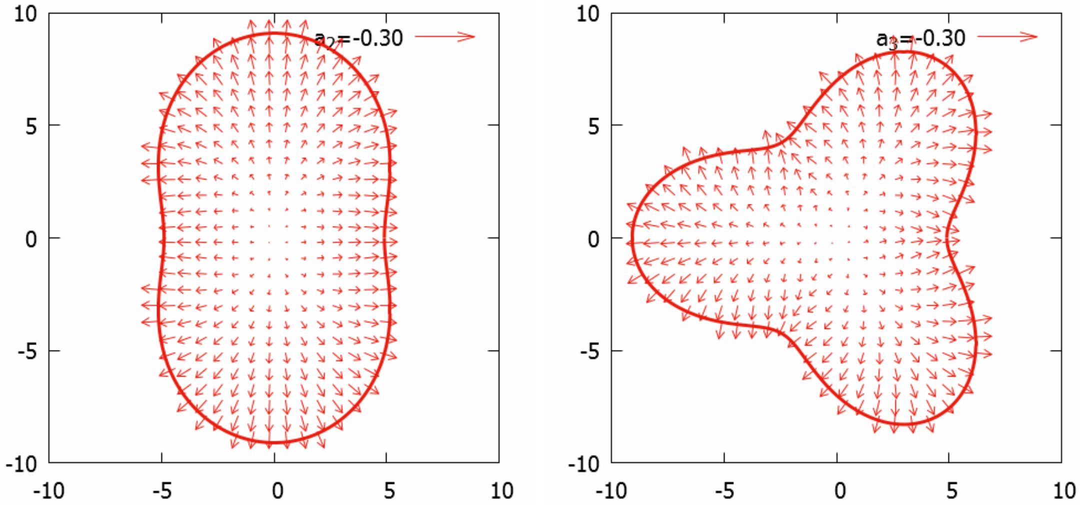
- Space anisotropy is characterized by Fourier series of the fireball radius

$$R(\theta) = R_0(1 - a_2 \cos(2(\theta - \theta_2)) - a_3 \cos(3(\theta - \theta_3)))$$

- Parametrisation is different than in Lisa-Retiere work [PRC70, 044907]



Space anisotropy

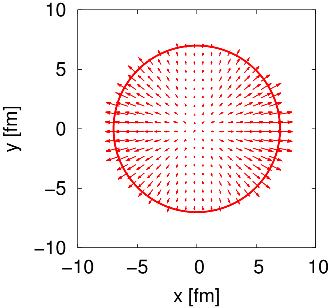


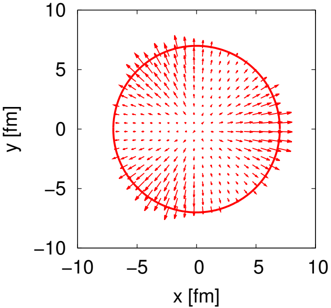
Flow anisotropy

- Flow anisotropy is characterized by Fourier series of the transverse rapidity

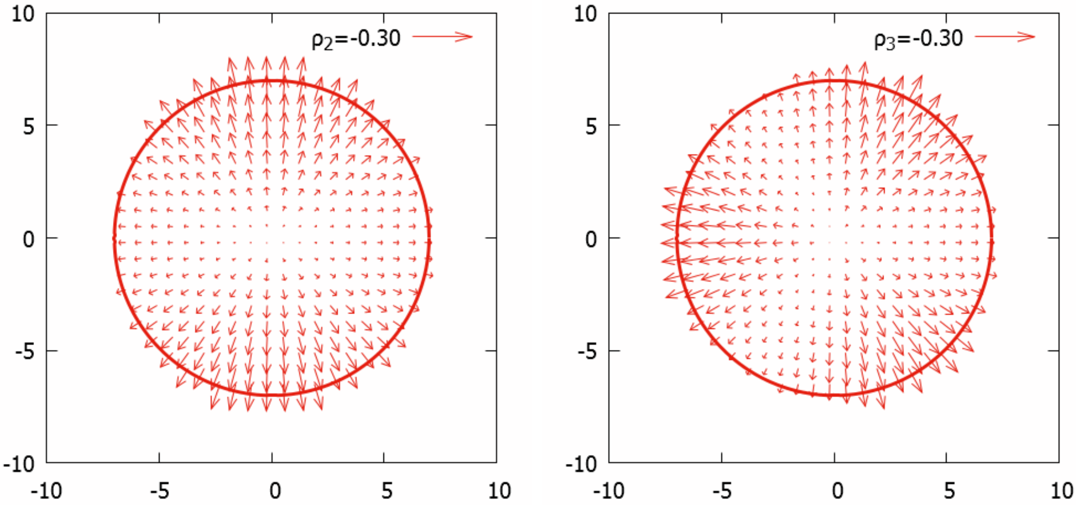
$$\rho(\bar{r},\theta_b) = \bar{r}\rho_0(1 + 2\rho_2\cos(2(\theta_b - \theta_2)) + 2\rho_3\cos(3(\theta_b - \theta_3)))$$

- θ_b is angle perpendicular to fireball surface





Flow anisotropy

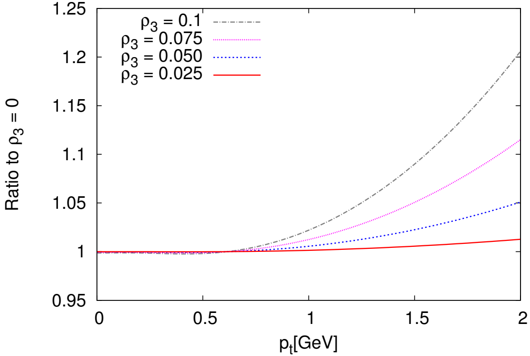


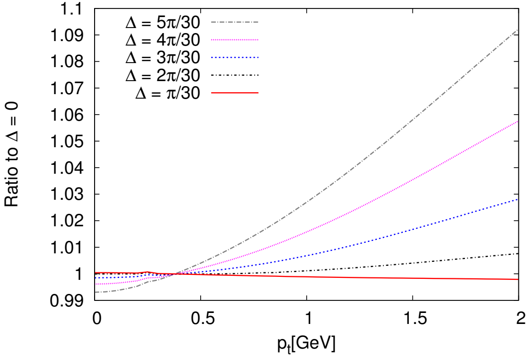
Momentum spectrum

- Single-particle spectrum can be calculated via

$$P(p_t,\phi) = \frac{\mathrm{d}^3 N}{p_t \mathrm{d}p_t \mathrm{d}Y \mathrm{d}\phi} = \int S(x,p)\mathrm{d}^4 x$$

- Flow anisotropy also influences azimuthally integrated spectrum





Fourier decomposition of spectrum

- Single-particle spectrum is azimuthally dependent and thus it can be decomposed into Fourier series

$$P(p_t,\phi) = \frac{1}{2\pi}\frac{\mathrm{d}^2 N}{p_t \mathrm{d}p_t \mathrm{d}Y}\bigg|_{Y=0}\left(1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \theta_n))\right)$$

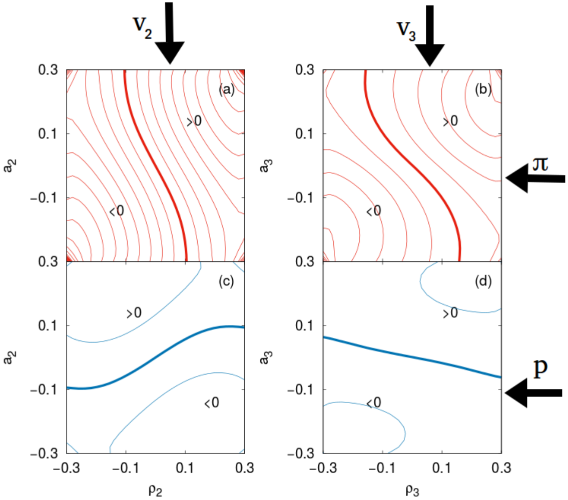
- Each coefficient can be calculated via formula

$$v_n(p_t) = \frac{\int_0^{2\pi} P(p_t,\phi)\cos(n(\phi-\theta_n))\mathrm{d}\phi}{\int_0^{2\pi} P(p_t,\phi)\mathrm{d}\phi}$$

- In an experimental analysis, one effectively takes an average over all possible values of $\Delta = \theta_3 - \theta_2$, so we have to add this averaging and integrate over Δ

Contour plots of coefficients v_2 and v_3

- There is ambiguity in determination of coefficients v_n
- We can get the same v_n with different combination of parameters a_n and ρ_n
- Model parameters are $T = 120$ MeV, $\rho_0 = 0.8$, $R_0 = 7$ fm, $\tau_0 = 10$ fm/c, $p_t = 300$ MeV/c



Increment (decrease) by 0.005

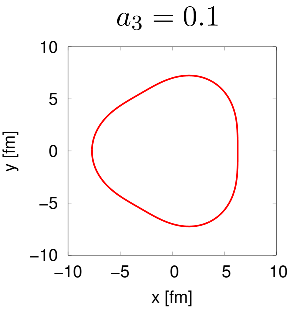
HBT radii

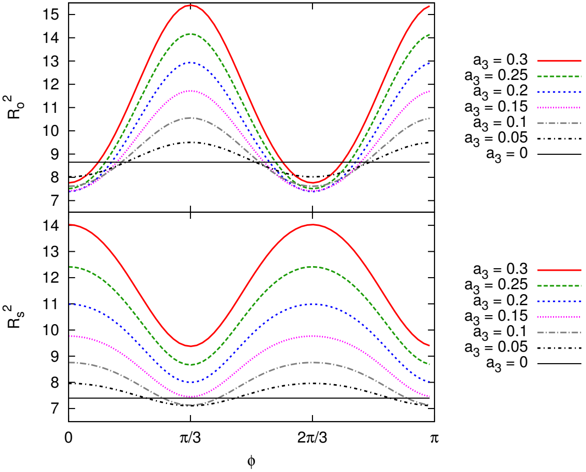
- Correlation radii characterize the homogeneity region of the fireball and are given via equations

$$R_o^2(K) = \left\langle (\tilde{x}_o - \beta_o \tilde{t})^2 \right\rangle (K)$$
$$R_s^2(K) = \left\langle \tilde{x}_s^2 \right\rangle (K)$$
$$R_l^2(K) = \left\langle (\tilde{x}_l - \beta_l \tilde{t})^2 \right\rangle (K)$$
$$R_{os}^2(K) = \langle \tilde{x}_o \tilde{x}_s \rangle (K) - \beta_o \left\langle \tilde{t} \tilde{x}_s \right\rangle (K)$$
$$R_{ol}^2(K) = \left\langle (\tilde{x}_o - \beta_o \tilde{t})(\tilde{x}_l - \beta_l \tilde{t}) \right\rangle (K)$$
$$R_{sl}^2(K) = \langle \tilde{x}_l \tilde{x}_s \rangle (K) - \beta_l \left\langle \tilde{t} \tilde{x}_s \right\rangle (K)$$

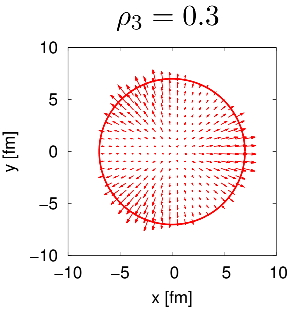
- where

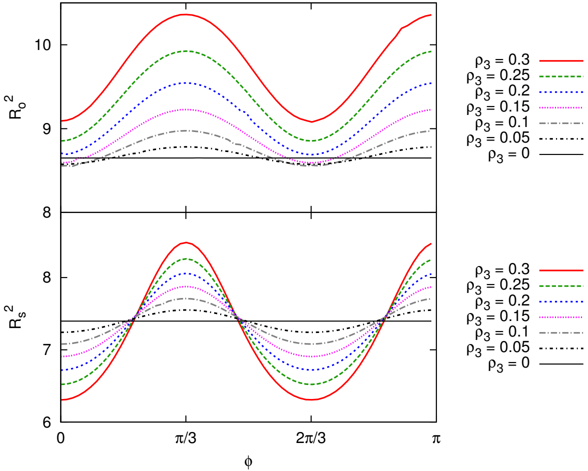
$$\tilde{x}_\mu = x_\mu - \langle x_\mu \rangle$$
$$\langle f(x) \rangle (K) = \frac{\int \mathrm{d}^4 x f(x) S(x, K)}{\int \mathrm{d}^4 x S(x, K)}$$


$a_3 = 0.1$
y [fm]
x [fm]
10
5
0
–5
–10
–10
–5
0
5
10


R_o^2
R_s^2
φ
0
π/3
2π/3
π
a_3 = 0.3
a_3 = 0.25
a_3 = 0.2
a_3 = 0.15
a_3 = 0.1
a_3 = 0.05
a_3 = 0

HBT radii for different ρ_3





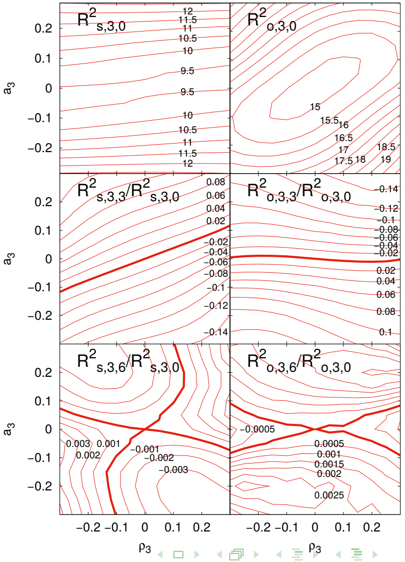
Oscillation amplitudes of HBT radii

- Fourier series of the correlation radii

$$R_i^2(\phi) = R_{i,3,0}^2 + \sum_{n=1}^{\infty} R_{i,3,n}^2 \cos(n(\phi - \phi_n))$$

- To avoid the trivial scaling with R_0, we divided all amplitudes by the zeroth-order term

- The third-order amplitude is mainly set by the spatial anisotropy parameter a_3



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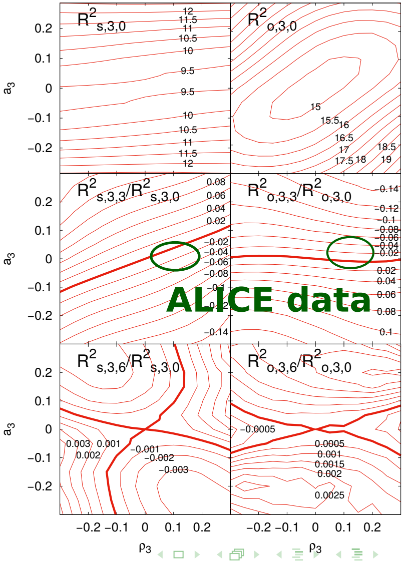


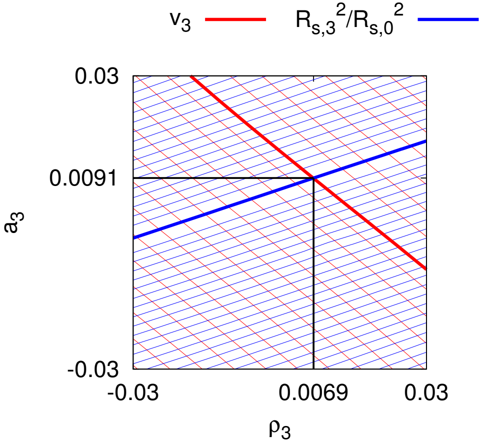
Qualitative ilustration of data analysis

- After measuring v_n and one of the correlation radii $R^2_{i,n}$ we can combine contour plots and get exact parameters of both anisotropies a_n and ρ_n
- We looked at data from PHENIX, because second order anisotropies have been analysed in these data, so we fixed model parameters and determined coefficients a_3 and ρ_3 from measured v_3 and $R^2_{s,3}/R^2_{s,0}$

[STAR] Phys.Rev. C71 (2005) 044906

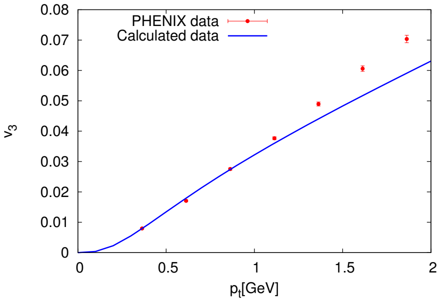
[PHENIX] Phys. Rev. Lett. 107 (2011) 252301

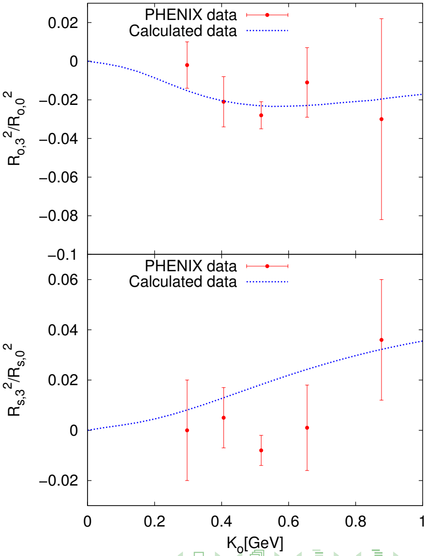
[PHENIX] Phys. Rev. Lett. 112 (2014) 222301



Qualitative ilustration of data analysis

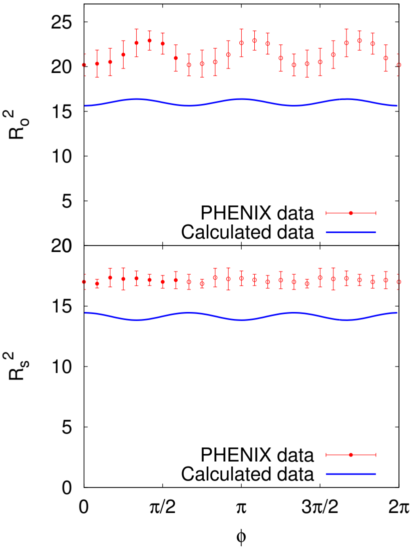
- Momentum dependence of Fourier coefficients calculated from Blast-wave model compared with data from PHENIX.





Qualitative ilustration of data analysis

- Azimuthal dependence of HBT radii
- Model parameters are $p_t = 530$ MeV, $\tau = 7.8$ fm/c, $\Delta\tau = 2.59$ fm/c, $R_0 = 11.4$ fm, $T = 98$ MeV, $\rho_0 = 0.98$
- Difference between data and calculations shows that we can't use parameters from one experiment and use them to fit results from another experiment



Conclusions

- We extended and generalized Blast-wave model into higher harmonics
- There is ambiguity in determination of a_n and ρ_n just from v_n or as HBT
- Parameters a_n and ρ_n can be uniquely determined only by combination of v_n and $R^2_{i,n}$ measurements
- Third order anisotropies generate higher-order coefficients v_n and $R^2_{i,n}$
- Reference: arXiv:1702.01735v1