Azimuthally sensitive femtoscopy

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Emission function

- Emission function is defined as probability, that a particle with 4-momentum p is emitted from spacetime point x
- Formally it is a Wigner phase-space density
- By integrating emission function through volume of fireball we get spectrum

$$P(p_t, \phi) = \frac{\mathrm{d}^3 N}{p_t \mathrm{d}p_t \mathrm{d}Y \mathrm{d}\phi} = \int S(x, p) \mathrm{d}^4 x$$

- We can decompose spectrum into Fourier series, where Fourier coefficients can be expressed as

$$v_n(p_t) = \frac{\int_0^{2\pi} P(p_t, \phi) \cos(n(\phi - \theta_n)) \mathrm{d}\phi}{\int_0^{2\pi} P(p_t, \phi) \mathrm{d}\phi}$$

Two-particle correlation function

- Correlation function is defined as ratio of two-particle spectrum and one-particle spectra
- We use correlation function in form

$$C(q,K) - 1 \approx \frac{|\int \mathrm{d}^4 x S(x,K) \exp(iqx)|^2}{\left(\int \mathrm{d}^4 x S(x,K)\right)^2}$$

  - $K = \frac{1}{2}(p_1 + p_2)$, $q = p_1 - p_2$

- Correlation function can be approximated by Gauss distribution

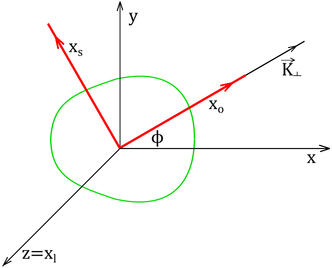
$$\begin{aligned} C(q,K) - 1 &\approx \exp\left(-q^\mu q^\nu \left\langle \tilde{x}_\mu \tilde{x}_\nu \right\rangle\right) \\ &= \exp\left(-R_o^2 q_o^2 - R_s^2 q_s^2 - R_l^2 q_l^2 - 2R_{os}^2 q_o q_s - 2R_{ol}^2 q_o q_l - 2R_{sl}^2 q_s q_l\right) \end{aligned}$$

  - where we used $q_0 = \vec{q} \cdot \vec{K} / K_0$

HBT radii

- HBT radii R_i give us information about size of homogeneity region of fireball

$$R_o^2(K) = \left\langle \left(\tilde{x}_o - \beta_o \tilde{t}\right)^2 \right\rangle (K)$$
$$R_s^2(K) = \left\langle \tilde{x}_s^2 \right\rangle (K)$$
$$R_l^2(K) = \left\langle \left(\tilde{x}_l - \beta_l \tilde{t}\right)^2 \right\rangle (K)$$
$$R_{os}^2(K) = \left\langle \left(\tilde{x}_o - \beta_o \tilde{t}\right) \tilde{x}_s \right\rangle (K)$$
$$R_{ol}^2(K) = \left\langle \left(\tilde{x}_o - \beta_o \tilde{t}\right) \left(\tilde{x}_l - \beta_l \tilde{t}\right) \right\rangle (K)$$
$$R_{sl}^2(K) = \left\langle \left(\tilde{x}_l - \beta_l \tilde{t}\right) \tilde{x}_s \right\rangle (K).$$



Blast-wave model

- This theoretical model is characterized by emission function

$$S(x,p)\mathrm{d}^4x = \frac{m_t\cosh(\eta - Y)}{(2\pi)^3}\mathrm{d}\eta\mathrm{d}x\mathrm{d}y\frac{\tau\mathrm{d}\tau}{\sqrt{2\pi}\Delta\tau}\exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)\exp\left(-\frac{p^\mu u_\mu}{T}\right)\Theta\left(1-\overline{r}\right)$$

- where

$$p_\mu u^\mu = m_t\cosh\rho\cosh(\eta - Y) - p_t\sinh\rho\cos(\phi - \theta_b)$$
$$\bar{r} = \frac{r}{R(\theta)}$$

  - θ_b is angle perpendicular to fireball surface
- Spatial anisotropy describes shape of the fireball

$$R(\theta) = R_0\left(1 - \sum_{n=2}^{\infty} a_n\cos\left(n(\theta - \theta_n)\right)\right)$$

- Flow anisotropy describes distribution of transverse rapidity

$$\rho(\overline{r},\theta_b) = \overline{r}\rho_0\left(1 + \sum_{n=2}^{\infty} 2\rho_n\cos\left(n(\theta_b - \theta_n)\right)\right)$$

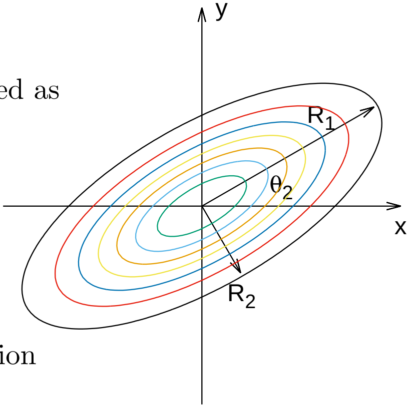
Gaussian emission function

- Consider Gaussian emission function

$$S(x,y) \propto e^{-ax^2-by^2+2cxy}$$

- Parameters a, b, c can be expressed as

$$a = \frac{\cos^2\theta_2}{2R_1^2} + \frac{\sin^2\theta_2}{2R_2^2}$$
$$b = \frac{\sin^2\theta_2}{2R_1^2} + \frac{\cos^2\theta_2}{2R_2^2}$$
$$c = -\frac{\sin 2\theta_2}{4R_1^2} + \frac{\sin 2\theta_2}{4R_2^2}$$



- Correlation function of this emission function is

$$C(q) - 1 = e^{-R_1^2(q_o\cos\theta_2 - q_s\sin\theta_2)^2 - R_2^2(q_o\sin\theta_2 + q_s\cos\theta_2)^2}$$

Averaging of correlation function

- In experiment we sum correlation functions over many events $\Rightarrow$ this can affect the shape of correlation function
- We consider uniform distribution of angle θ_2
- Averaged correlation function can be computed as

$$\int dR_1 \int dR_2 \int d\theta_2 (C(q) - 1)$$

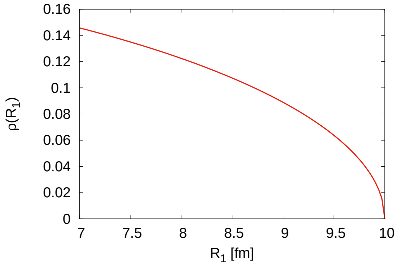
- Resulting function can be fit by Lévy distribution

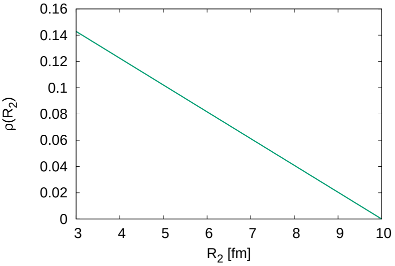
$$C(q) - 1 \approx \exp(-|qR|^{\alpha})$$

Distribution of sizes

- Size of the fireball can be distributed
  - uniformly
  - nonuniformly, depending on impact parameter (which has linear probability density) via equations

$$R_1 = \sqrt{R^2 - \frac{b^2}{4}} \qquad\qquad R_2 = R - \frac{b}{2}$$





Averaging of correlation function

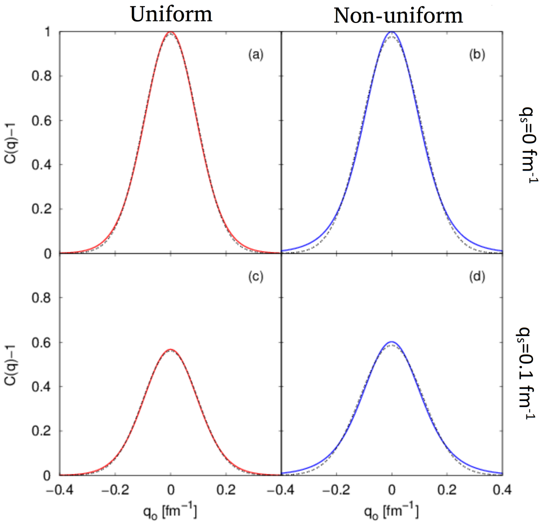
- To quantify, how much are these functions different from Gaussian function, we use Lévy distribution

(a) $\alpha = 1.8659$

(b) $\alpha = 1.8661$

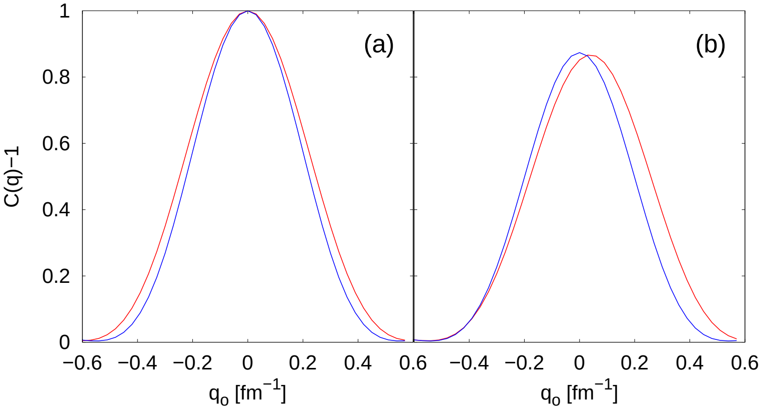
(c) $\alpha = 1.7052$

(d) $\alpha = 1.6806$



Correlation function of blast-wave model

- We can also study influence of angle averaging in Blast-wave model
- Averaging changes the shape of the correlation function about 0.5%

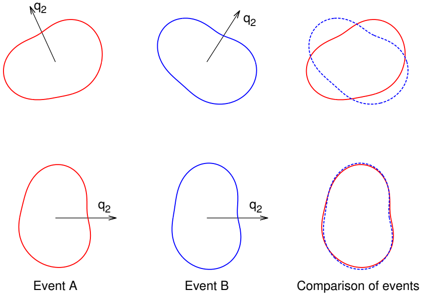


Femtoscopy with similar events

- We generate events with DRAGON (DRoplet and hAdron GeneratOr for Nuclear collisions) and AMPT (A Multi-Phase Transport)
- We sort events by its shape with Event Shape Sorting
- We calculate correlation function with CRAB (CoRrelation After Burner)
- By fitting correlation function we get correlation radii, so we can study their azimuthal dependence

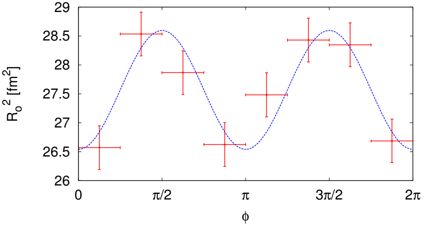
- Events may be similar, even if they do not seem at first glance
- We have to rotate all events to have the same value of vector

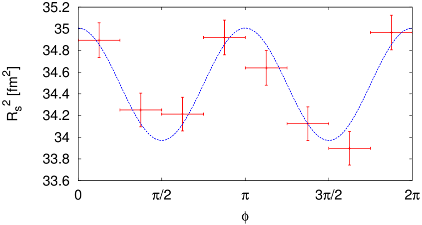
$$\vec{q_2} = \left(\sum \cos(2\phi_i), \sum \sin(2\phi_i)\right)$$



Azimuthal dependence of correlation radii

- Once we have rotated all events in one direction, second order anisotropy will sum up, while third order will be averaged out
- We can observe this in the resulting azimuthal dependence of correlation radii

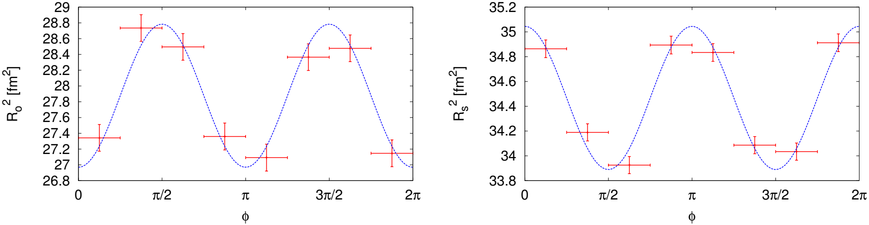




- 10 000 events generated via DRAGON

Azimuthal dependence of correlation radii

- Once we have rotated all events in one direction, second order anisotropy will sum up, while third order will subtracts
- We can observe this at resulted azimuthal dependence of correlation radii



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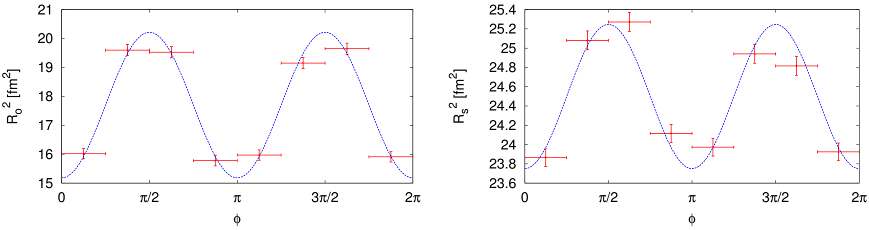
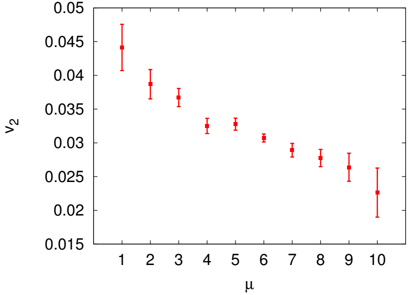
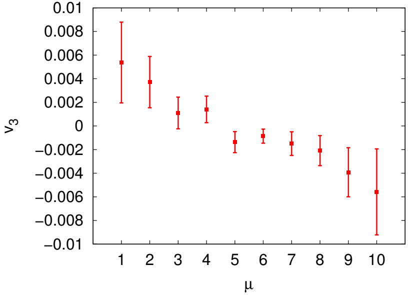


- 5 000 events generated via AMPT

Anisotropic flow in similar events

- After sorting events we split them into 10 classes
- We can calculate v_2 and v_3 from azimuthal distribution of particles for each class
- Evolution of these coefficients across classes shows us, how the average shape is changing

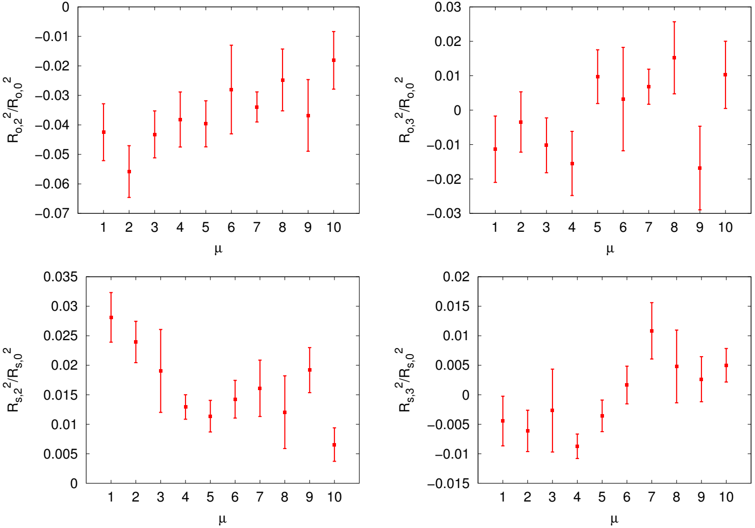




Femtoscopy of similar events

- In each class we can also obtain azimuthal dependence of correlation radii
- We can then see both second and third order anisotropies at the same time
- We can also see, how average shape evolves between classes
- Because of time-consuming nature of programs we have done this process just for sample of 10 000 events generated via DRAGON
- We decomposed R_o^2 and R_s^2 into Fourier series and calculated series' coefficients

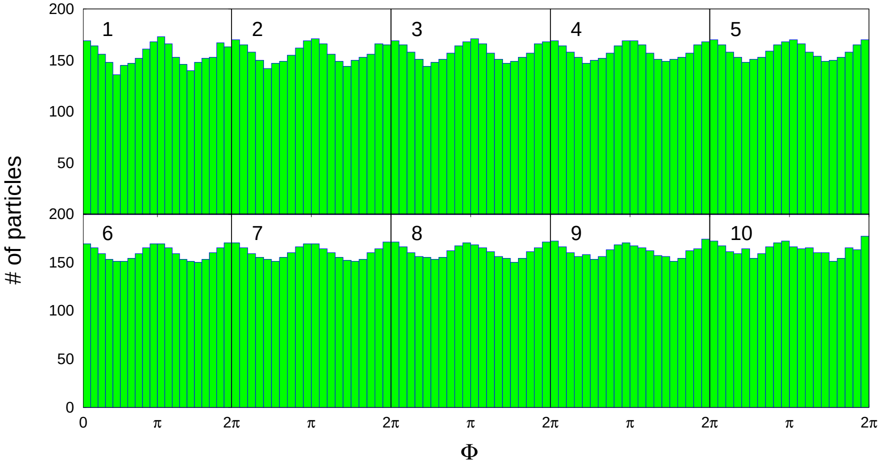
Correlation of similar events



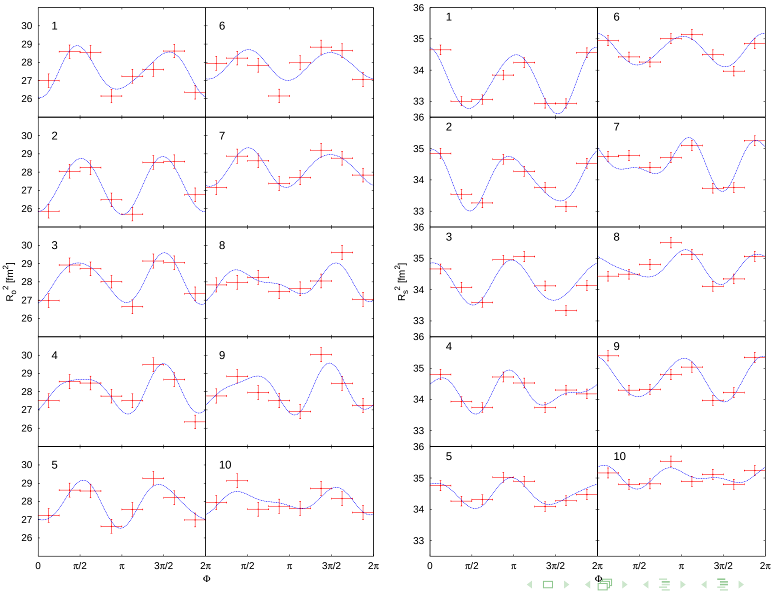
Summary and next research

- In the first part we have showed, how can averaging influence the shape of correlation function
- This means, that Gaussian correlation function does not imply Gaussian emission function
- In the second part we have showed, that thanks to sorting events and calculating correlation functions of similar events we can observe both second and third order anisotropies of correlation function at the same time
- Next research will continue with increased number of events to reduce uncertainties

Particle distributions in classes



Correlation of similar events



Phase evolution in classes